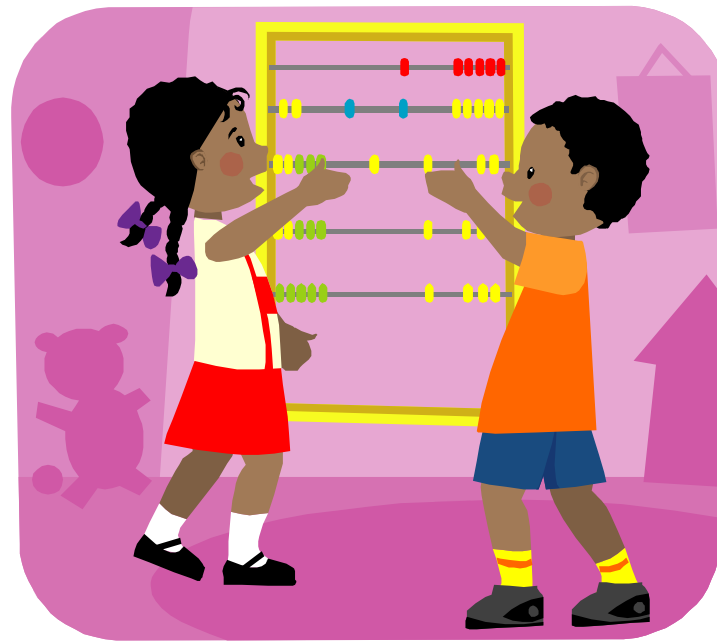


Pre-Calc Lesser Topics

Not as important for studying Calculus...

(These will, however, be on your final PreCalculus exam!)



Click when ready to proceed!

Solving Linear Systems

How would you solve this system?

$$ax + by + cz = d$$

$$fx + gy + hz = j$$

$$kx + my + nz = p$$



Put the coefficients into a 3x4 matrix:

| a b c d |

| f g h j | then 2nd quit, matrix math, rref

| k m n p |

| 1 0 0 answer for x |

and you should get: | 0 1 0 answer for y |

| 0 0 1 answer for z |

(Click when ready to continue)



Modifying known graphs

What happens to a graph if we add a positive number to the x term?



The graph slides to the left.



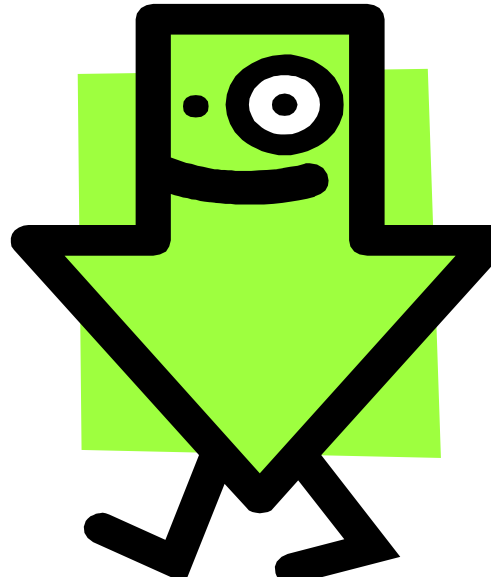
What if we subtract a
positive number from the
x-value?

The graph slides to the right.



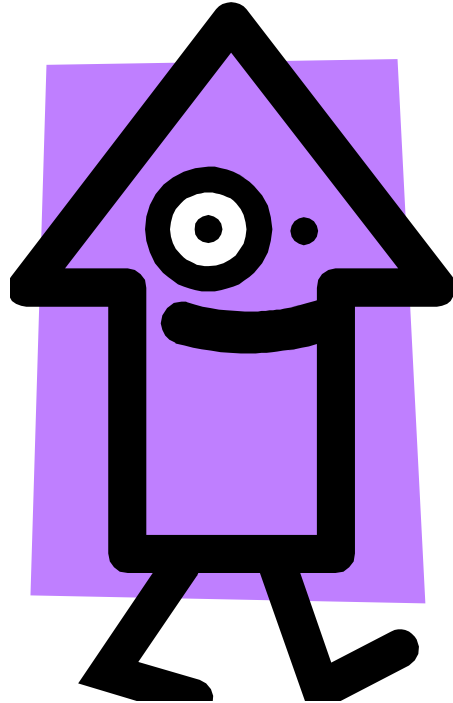
What about adding a positive to the y -value?

Yup...it's still counter intuitive, but consistent...the graph slides down.



Subtracting a positive from the y-value?

Yada yada yada...graph slides up.



What's the difference between the graph of $y = f(x)$ and $y = f(x) + C$?

The graph of $f(x)$ moves up if $c > 0$
and down if $c < 0$
Did ya get fooled?



It's a moving experience!

OK...click for the last two of these type of questions...

What if we multiply $f(x)$?
That is, what happens if $y = a \cdot f(x)$
and $a = 2$

$f(x) =$



What's he gonna look like?

He'll be twice as tall!



What if $a = 1/3$?

He's squished vertically to $\frac{1}{3}$ his
original height.
(Poor baby!)



OK...let's move on to the next topic....

What's the law of Sines?



The Law of Sines can help you solve any triangle as long as you have one side and the angle across from it, plus any other one measurement.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Click to continue.



What's the Law of Cosines?

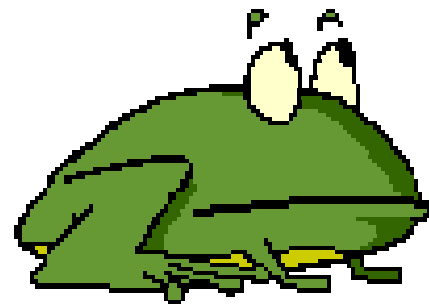


Ready?

Use the law of cosines if you have 2 sides and the angle between them OR all 3 sides but no angles.

(Use the Law of Sines otherwise, because it is easier!)

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

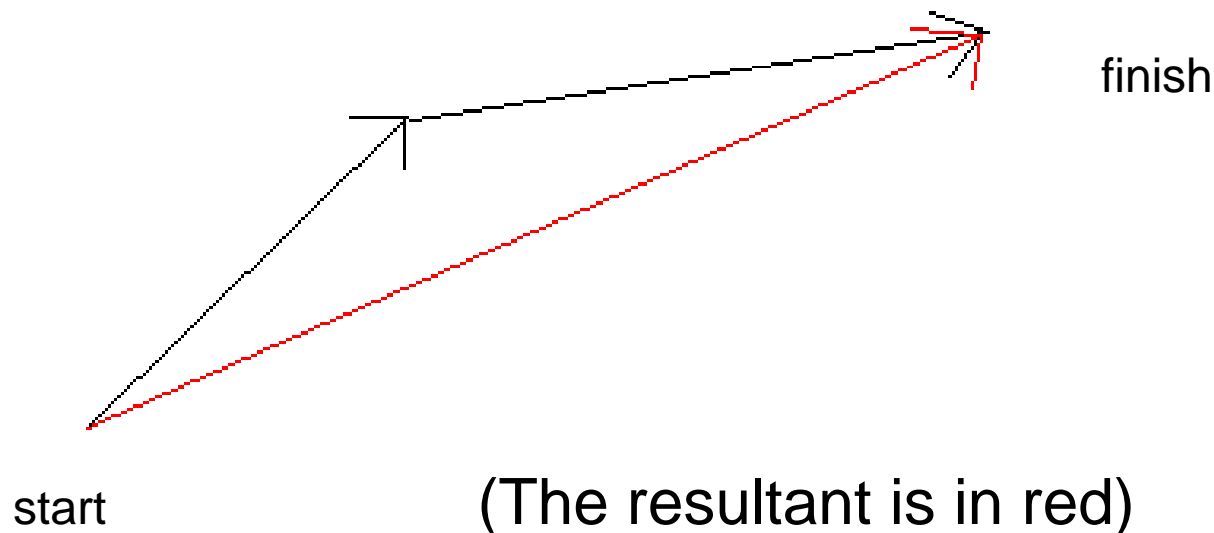


Ready to leap ahead to the next slide??

Vectors!

Q. How do you add them geometrically?

A. Slide one of them on top of the other and then draw the resultant from the tail of the first one to the head of the second one. (start to finish)



How do you add and subtract analytically?

(It's easy!)

Try these:

1. $\langle 3, 4, 5 \rangle + \langle -3, 4, 6 \rangle$

2. $\langle -4, 6, 10 \rangle - \langle 4, -2, -20 \rangle$

3. $(6i + 5j - 2k) - 2(5i - 8j - 6k)$

Did you write down your answers?



Here are the correct answers:

1. $\langle 0, 8, 11 \rangle$ or $8j + 11k$
2. $\langle -8, 8, 30 \rangle$ or $-8i + 8j + 30k$
3. $\langle -4, 21, 10 \rangle$ or $-4i + 21j + 10k$

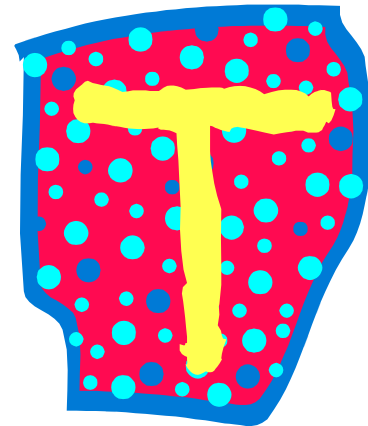
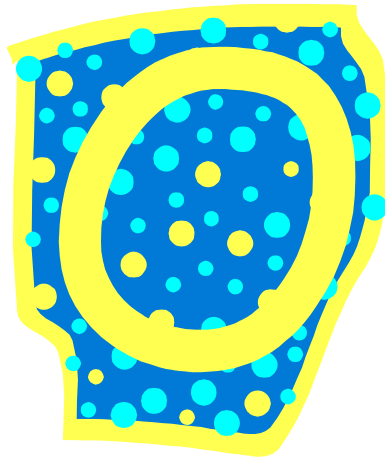
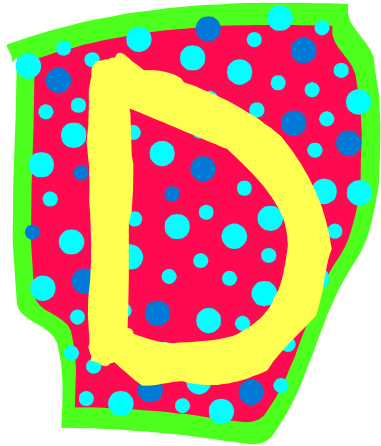
How'd ya do??

Let's try dot product
and cross product next.
Click at will!



What's the rule for dot products?

$$\langle a, b, c \rangle \cdot \langle d, e, f \rangle = ?$$



$a \cdot d + b \cdot e + c \cdot f =$ a single
number

Example:

$$\langle 2, 3, -1 \rangle \cdot \langle 0, 5, -3 \rangle = 2 \cdot 0 + 3 \cdot 5 + -1 \cdot -3 = 18$$

How do you compute the cross product?



Use a determinant!

$$\begin{vmatrix} i & j & k \\ a & b & c \\ d & e & f \end{vmatrix}$$

Do you remember how to evaluate it?

Try this one and then check your answer on the next slide:

$$\langle 1, 2, 0 \rangle \times \langle -3, 0, 4 \rangle = ?$$

Did you get it right?

$$\begin{array}{c} \begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 2 & 0 & 1 & 2 \\ -3 & 0 & 4 & -3 & 0 \end{array} \\ \begin{array}{l} -6k + 0i + 4j \\ 8i + 0j + 0k \end{array} \end{array} = \text{bottom} - \text{top} = 8i - 4j + 6k$$

Click to go on to parametric functions.



Parametric Equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

That is, the (x,y) on the graph is defined by using a third variable called the parameter.

We usually use t (which you can think of as time).

For example, make a t -chart and then graph:

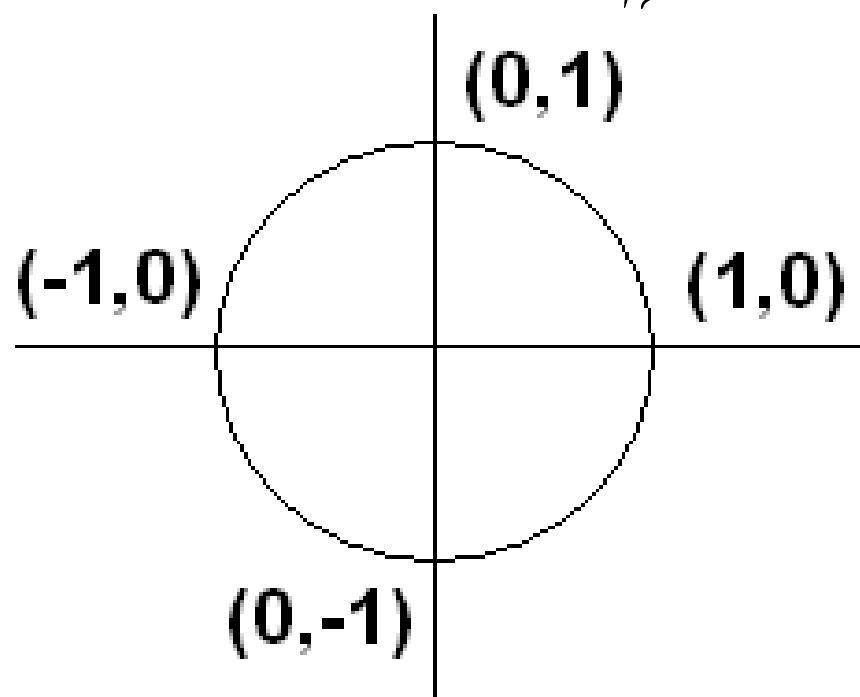
$$x(t) = \cos t$$

$$y(t) = \sin t$$

How did you do?



t	x	y
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0
$\frac{5\pi}{4}$	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
$\frac{3\pi}{2}$	0	-1
$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
2π	1	0



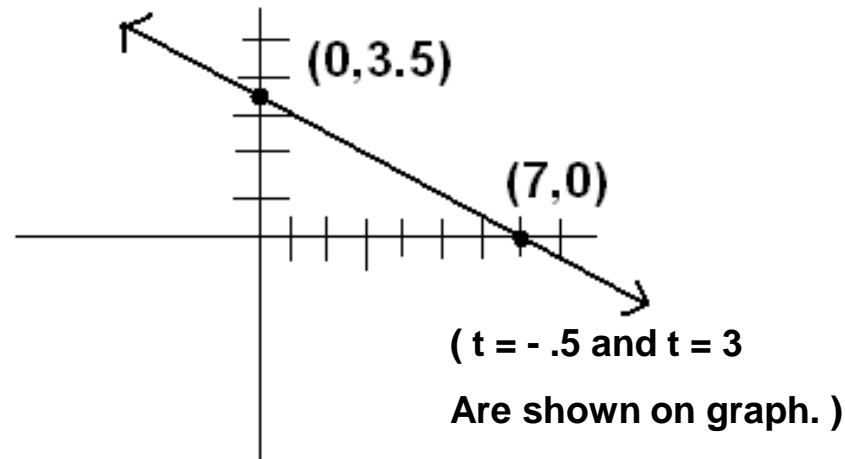
Try changing to “par” MODE and graphing this on your calculator also.

Note: if x and y are both linear functions in t , then the graph will be a line also.

Example:

$$x(t) = 2t + 1$$

$$y(t) = 3 - t$$



Graph it like we did the last one, or combine into a regular linear equation by solving for x in terms of t and then substituting:

$$t = \frac{x-1}{2} \quad \longrightarrow \quad y = 3 - \frac{x-1}{2} = -\frac{1}{2}x + 3.5$$

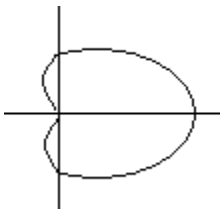
Which is clearly a line!!

Polar graphs are pretty!

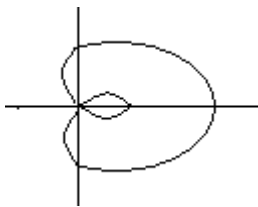
Polar graphs are difficult to draw by hand, but Pre-Calc students need to be well acquainted with the most common shapes, proficient in using their calculators and copying their graphs accurately onto paper.

The following (as well as circles) are the most usual shapes:

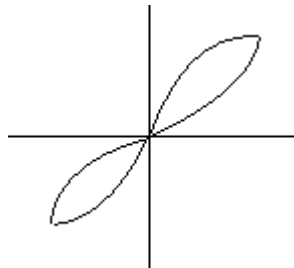
cardioid



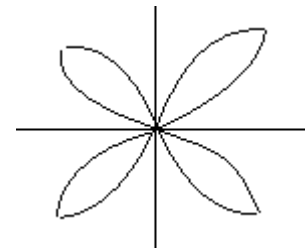
limaçon



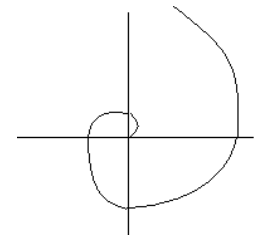
lemniscate



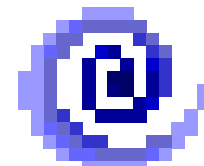
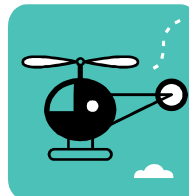
rose



spiral



Make sure your cardioids and limaçons show all the parts of the curves clearly in all 4 of the quadrants. Intercepts should also be accurate. Rose petals all meet at the origin but never overlap elsewhere.



Graph these on your calculator.
Make the window an appropriate size so that your graph
just fills the screen. What are the shapes?

$$r = 5 \sin 3\theta$$

$$r = 2 + 6 \cos \theta$$

$$r = 4 - 3 \sin \theta$$



Answers are on the next slide.

Answers:

$r = 5 \sin 3\theta \rightarrow$ 3 leaf rose – window size 5 or 6

$r = 2 + 6 \cos \theta \rightarrow$ limaçon – window size 8 or 9

$r = 4 - 3 \sin \theta \rightarrow$ cardioid – window size 7 or 8

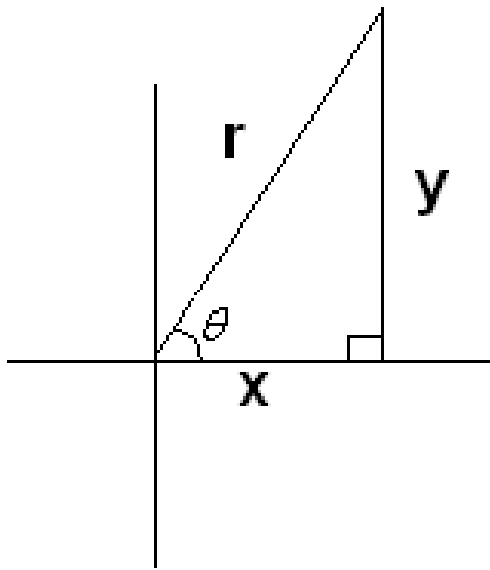
Having trouble?

Did you set your MODE to pol?

Are you in radian MODE?



Converting from polar to rectangular and vice versa uses the same basic formulas as vectors and complex numbers in the Argand plane. So memorize them 3 times...ha ha.



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

(Or add π if the original point is in Quadrants 2 or 3)

Click when ready for some examples.



Changing a polar point to rectangular form:

$$\left(3, \frac{\pi}{3}\right)$$

$$x = 3 \cos \frac{\pi}{3} = \frac{3}{2}$$

$$y = 3 \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{2} \quad \text{Answer: } \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$$

Changing a rectangular point to polar form:

$$(3, -2)$$

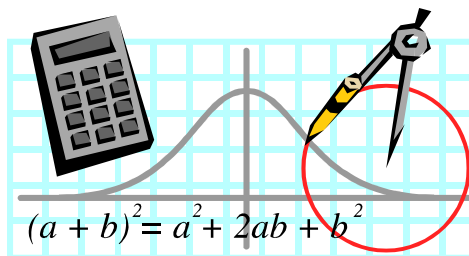
$$r = \sqrt{9 + 4} = \sqrt{13}$$

$$\theta = \tan^{-1}\left(\frac{-2}{3}\right) \approx -33.7^\circ \quad (\text{But the point is in Quadrant 2.})$$

$$\text{Answer: } (\sqrt{13}, -33.7^\circ + 180^\circ) = (\sqrt{13}, 146.3^\circ)$$

Changing equations necessitates some algebra.

(see next slide for some examples)



Changing from rectangular to polar is straight forward.

Example:

$$x^2 + y^2 = x - y$$

$$(r \cos \theta)^2 + (r \sin \theta)^2 = r \cos \theta - r \sin \theta$$

$$r^2 (\cos^2 \theta + \sin^2 \theta) = r(\cos \theta - \sin \theta)$$

1 ←

Now divide both sides by r. (BTW: Did you see a short cut?)

$$r = \cos \theta - \sin \theta$$

Changing to rectangular is trickier...you need to see x's and y's in polar form, and sometimes multiply by r or r squared to create what's needed.

Example: $\sin \theta + \cos \theta = r$

$$r \sin \theta + r \cos \theta = r^2 \quad \text{Multiply by } r$$

$$y + x = x^2 + y^2 \quad \text{Recognize the formulas for } x \text{ and } y \text{ and } r \text{ squared.}$$

Introducing Mr. “i”

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i$$

Every 4 powers of i repeat ...so...
for i^n , just divide n by 4 and use the remainder.

Example:

$$i^{234764975} = ? \quad \begin{array}{r} 58691243 \text{ R3} \\ 4 \overline{)234764975} \end{array} \quad \text{so} \quad i^{234764975} = i^3 = -i$$

Complex Numbers

Complex numbers include:

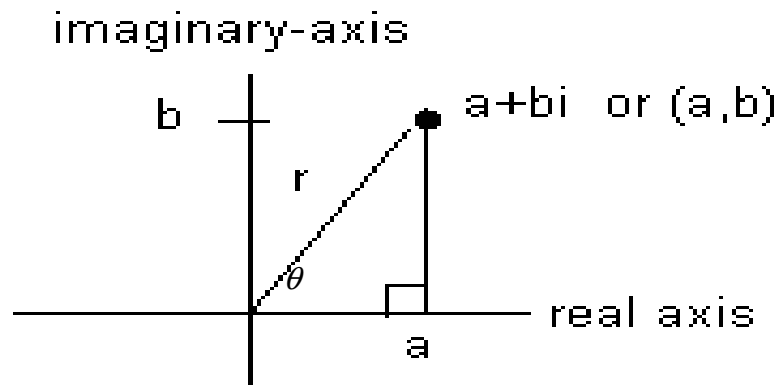
Real Numbers (they live on the horizontal number line)

Pure imaginary numbers (real multiples of i which live on the vertical imaginary-axis)

Complex numbers are reals, imaginaries and the sums of reals + imaginaries of the form:

$a + bi$ (rectangular form) or (r, θ) where $r = |a + bi| = \sqrt{a^2 + b^2}$
and θ is defined the same way as for polar form and vectors.

We can graph complex numbers on a plane called the Argand Plane



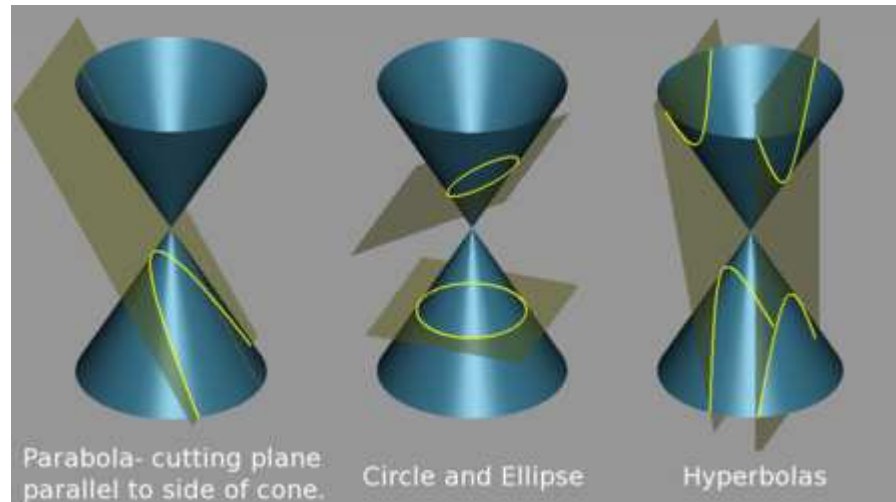
If $a = 0$, then the number is imaginary and it is graphed on the imaginary (vertical) axis.

If $b = 0$, then the number is real (graphed on the real/horizontal axis)

...but they are all Complex numbers.

Conics!

Three types:
Ellipses, Hyperbolas and Parabolas.
(circles are a subset of the ellipses.)



(Picture copied from Wikipedia)

The general formula for all conics is:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

The conics that most texts deal with are always parallel
to the x and/or y-axes.

This will be the case if and only if $B = 0$

(If B is NOT 0 then the conic is tilted and that's much harder to deal with.)

Let's look at the standard form for each conic because that's the form that's useful.
To change from general form to standard form necessitates completing the square.

Hopefully you know how to do that already.

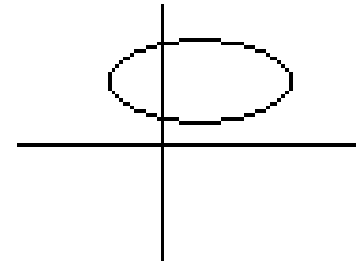
Circles

$$(x - h)^2 + (y - k)^2 = r^2$$

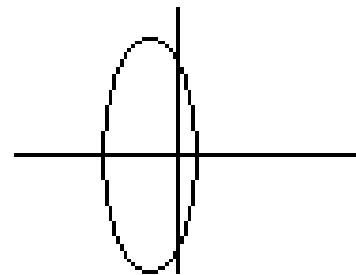
The center is (h,k) and the radius is r . Circles are superdeedoooper easy!

Ellipses!

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$



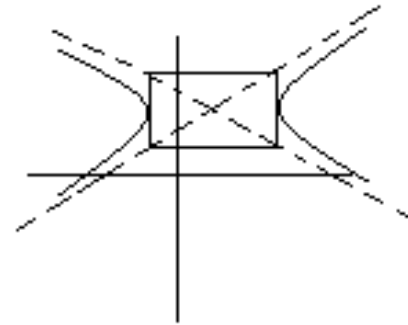
$$a^2 - b^2 = c^2$$

Remember that the major axis is $2a$ units long and the minor axis is $2b$ units long and that c is the distance from the center (h,k) to the foci.

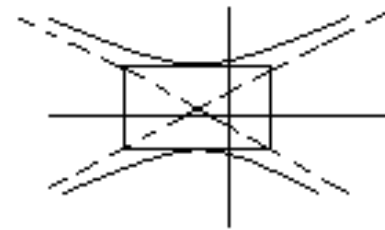
Also...in ellipses, a is the largest of the three letters: a , b and c .

Hyperbolas!

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



$$a^2 + b^2 = c^2$$

Remember that c is still the distance from the center (h,k) to the foci, but this time $c > a$. So now c is the biggest number. The term in front of the minus sign (first term) determines the position of the hyperbola and the a value.

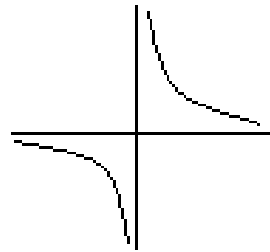
The equations of the asymptotes are:

$$(y-k) = \pm \frac{\Delta y}{\Delta x} (x-h)$$

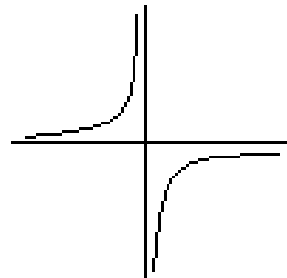
Special rotated hyperbolas!

When there is **only** an xy term in the general equation of a conic, then the result is a hyperbola which is rotated 45 degrees.

$$xy = k^2$$



$$xy = -k^2$$

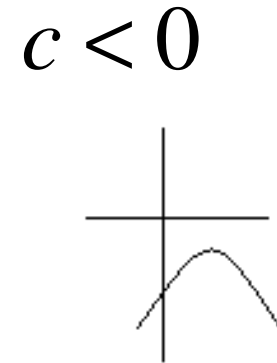
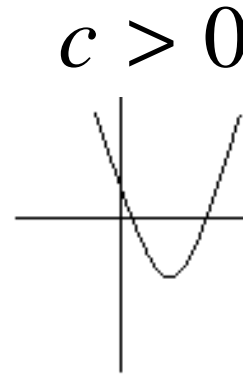


The vertices are at (k,k) , $(-k,-k)$ and $(-k,k)$, $(k,-k)$ respectively.

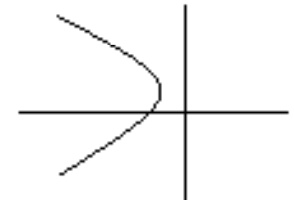
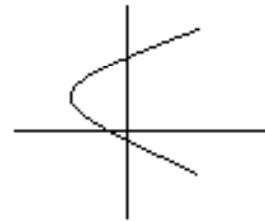
The length of the major axis connecting the vertices is $2\sqrt{2}k$.

Parabolas

$$(x - h)^2 = 4c(y - k)$$



$$(y - k)^2 = 4c(x - h)$$



Note: when c is positive the parabola looks like a letter of the alphabet (U or C).

If y is squared, then the parabola is sideways.

Also, c is the directed distance from the vertex to the focus and $|4c|$ is the width across the parabola at the focus (latus rectum).

The axis of symmetry is the line that splits the parabola down the middle and the directrix is a line $-c$ units from the focus perpendicular to the axis of symmetry

Eccentricity

The eccentricity of a conic gives a sense of its shape and type. For ellipses and hyperbolas:

$$e = \frac{c}{a}$$

When $c = 0$, there are no foci, just a center and therefore the conic is a circle.

As e grows from zero to one, the circle stretches out, elongating until $e = 1$

When $e = 1$, the shape “explodes” into a parabola.

When $e > 1$, then $c > a$ and the shape becomes a skinny hyperbola. It’s skinny because near 1, c is near a which makes b incredible small.

As e gets bigger and bigger the hyperbola gets wider and wider.

A great web site to see the effect of changing e is at:

<http://www.mathwords.com/multimedia/Conics%20defined%20by%20eccentricity.htm>

How to Rotate any conic!

Given: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

We need to somehow eliminate the xy term in order to get an equation in standard form – the form needed to graph the conic.

We use the following formulas to create such an equation:

$$\tan(2\theta) = \frac{B}{A - C}$$

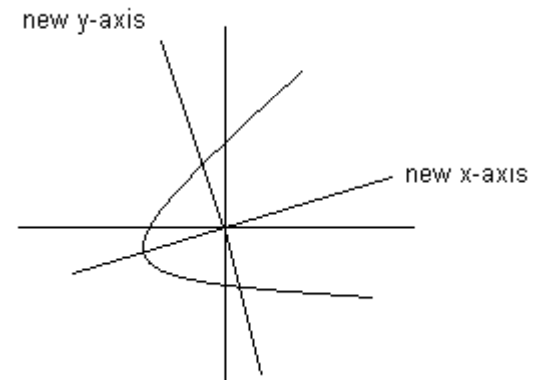
$$A' = A \cos^2 \theta + B \sin \theta \cos \theta + C \sin^2 \theta$$

$$(B' = B \cos 2\theta - (A - C) \sin 2\theta = 0)$$

$$C' = A \cos^2 \theta - B \sin \theta \cos \theta + C \sin^2 \theta$$

$$E' = -D \sin \theta + E \cos \theta$$

$$F' = F$$



That will give you a new equation w/o an xy term which you can graph on a set of axes rotated θ degrees from the original x and y -axes.

$$A'x^2 + C'y^2 + D'x + E'y + F' = 0$$

Series

A series is any sum of numbers, but they are NOT very interesting unless they go on forever. (Infinite series)

For example: $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$

(which we would need calculus to prove)

In PreCalculus, we only study 2 types of series: arithmetic and geometric, which we will explore on the next few slides.



Arithmetic Series

Each term is a fixed amount (d – called the “common difference”) bigger than the previous term.

For example: $1 + 4 + 7 + 10 + 13 + 16 + \dots$

Here, $d = 3$. The difference between each term and the previous term is 3.

We usually refer to the terms as a_n 's. That is, in the above example, $a_1 = 1$, $a_2 = 4$, $a_3 = 7$ etc. And $a_n = 1 + 3(n-1)$ = the first term plus n minus one d 's.

If we stop adding at the n th term, we call the sum a partial sum. In this example: $S_1 = 1$, $S_2 = 1 + 4 = 5$, $S_3 = 1 + 4 + 7 = 12$ etc.

Formulas:

$$a_n = a_k + (n - k)d$$

$$S_n = \sum_{k=1}^n [a_1 + (k - 1)d] = \frac{n}{2}(a_1 + a_n)$$

Please note that arithmetic series just keep growing, so we only examine finite arithmetic series – not very exciting.

An infinite arithmetic series is said to ***diverge***.

Geometric Series

Each term is a fixed amount, r , **times** the previous term.

This fixed amount, r , is called the common ratio because if you divide any term by the previous term (i.e. form a ratio) you will get r .

Example: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

In the example $r = \frac{1}{2}$ because each term is $\frac{1}{2}$ **times** the previous term.

Formulas: $a_n = a_k r^{n-k}$

$$S_n = \sum_{k=1}^n a_1 r^{k-1} = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S = \frac{a_1}{1 - r}$$

These formulas all assume that a_1 is the first term. If the first term is a_0 , then the formulas will need adjusting.

IMPORTANT! The infinite sum formula, S is only valid if $-1 < r < 1$

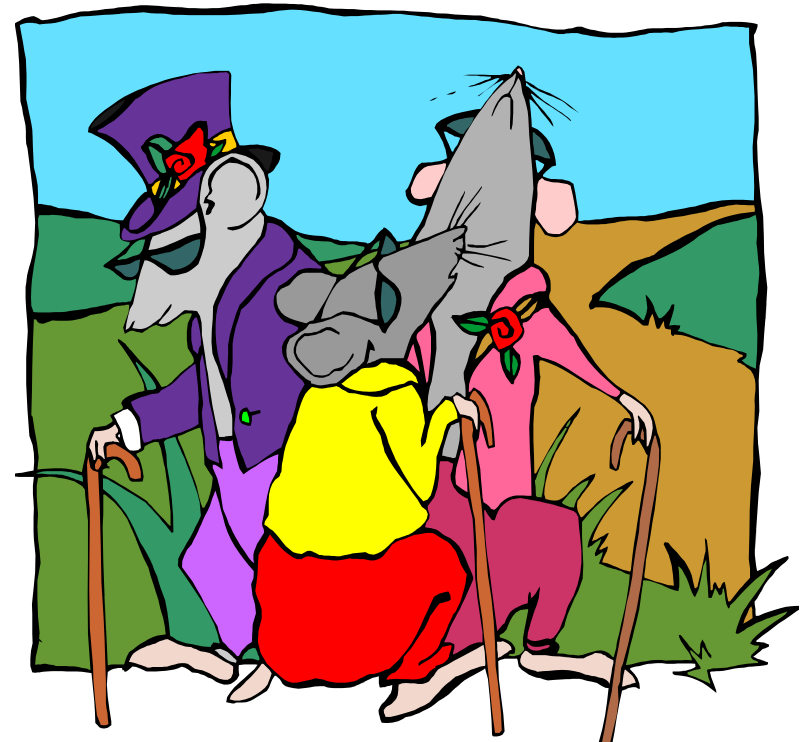
(Otherwise the terms get bigger and bigger...the series diverges.)

Time to sing a song!!!

(to the tune of *Three Blind Mice*)

I love math!
Math loves me!
Oh how smart,
I must be,
I love to do my Pre-Calculus,
I'm so good that I rarely miss,
Conics and series and logarithms,
Yes, I love math!

Ok...back to work now!



Learning How to Count

We all think we learned how to count in kindergarten, but counting large numbers of items or counting in tricky situations is difficult even for wise old educated people. Read the examples below.

There are 5 students running for 11th grade class president, 6 running for vice president, 4 running for secretary and 3 running for treasurer. How many different results are possible?

How many different 5 card poker hands can be dealt from a pack of 52 cards?

How many three digit numbers can be formed using only a 1,2,3 and 4?

(a) if digits can be repeated (b) if digits cannot be repeated



The Counting Principle

If there are m ways to do a first task and n ways to do a second task then there are mn ways to do both tasks.

Example: I have 6 pairs of pants and 9 shirts. How many outfits can I wear?

Task #1 – choose pants: 6 ways to do that

Task #2 – choose shirt: 9 ways to do that

Result: $6 \times 9 = 54$, so I have 54 possible outfits.



Answers to previous questions:

(using the counting principle)

There are 5 students running for 11th grade class president, 6 running for vice president, 4 running for secretary and 3 running for treasurer. How many different results are possible?

A: $5 \times 6 \times 4 \times 3 = 360$

How many different 5 card poker hands can be dealt from a pack of 52 cards?

A: $52 \times 51 \times 50 \times 49 \times 48 = 311,875,200$

How many three digit numbers can be formed using only a 1,2,3 and 4?

(a) if digits can be repeated

A: $4 \times 4 \times 4 \times 4 = 256$

(b) if digits cannot be repeated

A: $4 \times 3 \times 2 \times 1 = 24$

Combinations and Permutations

We use **combinations** if a different order of choosing gives the same result.

Example: I choose Jean, Bob, Fred and then Ethel to work as a team. I choose Fred, Ethel, Jean and then Bob to work as a team. Either way, it's the same team.



We use **permutations** when a different order means we get a different result.

Example: I choose Jean, Bob, Fred and then Ethel for president, vice president, secretary and treasurer in that order. I choose Ethel to be president, Fred as vice president, Jean to be secretary and then Bob as treasurer. Now I have two different results...

What if Bob is an embezzler?



Permutations

$${}_n P_r = \frac{n!}{(n-r)!} = \frac{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{(n-r)(n-r-1)\cdots 3\cdot 2\cdot 1} = n(n-1)(n-2)\cdots (n-r+1)$$

Example: A certain lock has numbers 1 through 39. A combination (which should be called a permutation since order makes a difference) consists of one number clockwise followed by a full turn counter clock wise to a second different number and finally clockwise to a third distinctly different number. How many different combinations are possible?

$$\text{Answer: } {}_{39}P_3 = \frac{39!}{(39-3)!} = \frac{39\cdot 38\cdot 37\cdot 36\cdot 35\cdots 3\cdot 2\cdot 1}{36\cdot 35\cdots 3\cdot 2\cdot 1} = 39\cdot 38\cdot 37 = 54,834$$

Note: Your lock probably includes 0 and allows repeated numbers like: 25-25-25.
In that case the number of possible “combinations” is: $40 \times 40 \times 40 = 64,000$



Combinations



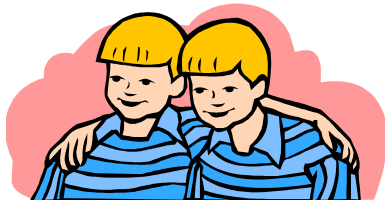
If order doesn't matter, then there are fewer possibilities. For example abc, acb, bac, bca, cab and cba are all the same result. So if, for example, we are selecting three objects, we will need to divide the number of possible permutations by three factorial.

Formula: ${}_n C_r = \frac{n!}{(n-r)!r!}$

Note that the formula is the same as for permutations except we are dividing by r factorial.

Example: How many ways can I select a prom committee of 5 students from 23 who have applied?

$${}_{23}C_5 = \frac{23!}{(23-5)!5!} = \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18!}{18! \cdot 5!} = \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 23 \cdot 11 \cdot 7 \cdot 19 = 33,649$$



Probability



Probability (defined mathematically) is the number of ways the desired event can occur divided by the total number of ways the event can occur where ***each way counted is equally likely***.

This ***equally likely*** requirement is often the sticking point for students and takes quite a bit of thought.

For example: Here are the four equally likely 2 child possibilities:

An older girl and a younger a boy, an older boy followed by a girl, two girls and finally two boys.

Many people think that there are only three equally outcomes, namely 2 boys, 2 girls or a boy and a girl...this is incorrect.

Try simulating families by tossing two pennies. Record a head as a girl and a tail as a boy and you will quickly see that one of each sex happens twice as often as two girls or two boys. Or toss a quarter and a penny and let the older child be represented by the quarter. There is nothing like an actual simulation to help you “see” the real probabilities. Intuition in these matters is often incorrect.



Here's another example...

I have a horrible disease and I ask my doctor, "If I take your treatment, what's the probability I will live?"

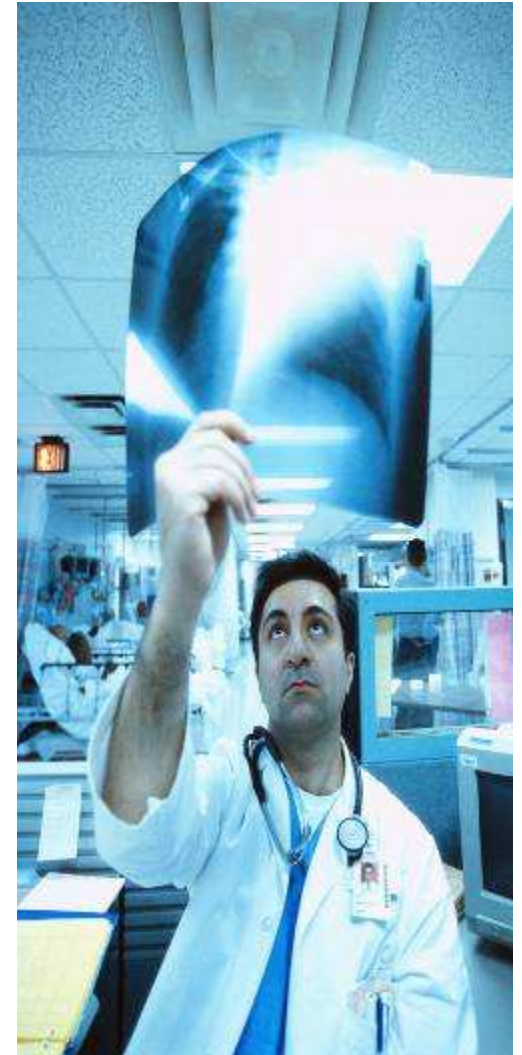
He says "50-50, because either you will live or die."

Obviously this doctor has forgotten his high school mathematics!! I really doubt that these two events are equally likely. I'm hoping that the treatment has a good chance of curing me – not just half a chance.

A better way of helping me see the reality of my situation is to use an empirical probability:

$$\frac{\text{number of patients he has treated for this disease that lived}}{\text{total number of patients he has treated for this disease}}$$

If, for example the number of such patients is quite large, this ratio will give me a pretty good idea of my chances!



Sample Spaces

The easiest way to deal with probability problems is to list the sample space of equally likely events and then count the number of successes.

If the sample space is too large to list, then you will need to abbreviate it, or find clever ways of counting all the possibilities.

In any event, beginning to list the possibilities, even if you can't complete the entire list, will give you insight into the problem.

Example: What is the probability of 2 or more heads when you flip four coins?

The Counting Principle says there should be $2 \times 2 \times 2 \times 2 = 16$ possible outcomes.

(four events – each with 2 possible outcomes)

So we want to make sure we list all 16...do so in an orderly fashion!

HHHH	HHHT	HHTT	HTTT	TTTT
	HHTH	HTHT	THTT	
	HTHH	THHT	TTHT	
	THHH	HTTH	TTHH	
		THTH		
		TTHH		

(all heads) (3 heads) (2 heads) (one head) (no heads)

$$\text{Probability} = \frac{11}{16}$$



Simple Probabilities

(one event)

Try these:

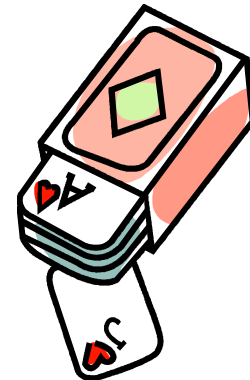
1. The probability of rolling a prime number when tossing a single die.
2. The probability of drawing a king at random from a standard deck of cards.
3. The probability of picking a red bead at random from a jar containing 60 red beads, 40 green beads and 30 yellow beads.
4. The probability of drawing a face card or a heart from a standard deck.



Answers!

1. 2, 3 and 5 are prime...so $3/6$ or $1/2$
2. There are 4 Kings...so $4/52$ or $1/13$
3. There are 60 red beads and 130 total...so $60/130$ or $6/13$
4. This one is tricky! Don't count the King, Queen and Jack of hearts twice!

$$P = \frac{4(3) + 10}{52} = \frac{22}{52} = 11/26$$



Harder Probabilities

(compound events)

The rule of thumb for probabilities when more than one event is involved is to multiply the probabilities when the problem infers “and”, but add the probabilities when the problem infers “or”. In the latter case, you need to watch out for overlap!

Example:

1. What’s the probability that when throwing 2 dice, they both are 4’s?

Answer: That infers: the first die is 4 **and** the second die is 4

$$P = 1/6 \times 1/6 = 1/36$$

2. What’s the probability that one die is a four and the other one is a prime number?

This one is tricky because it infers that either the first die is a 4 **and** the second is a prime **or** the first one is a prime **and** the second one is a 4:

$$P = 1/6 \times 1/2 + 1/2 \times 1/6 = 1/6$$



Pascal's Triangle

Pascal's triangle is useful because all the entries are combinations.
But you have to start counting at zero.

Row zero				1							
Row one			1		1						
Row two			1		2		1				
Row three		1		3		3		1			
Row four		1		4		6		4		1	
Row five	1		5		10		10		5		1
Etc.											

So if you want ${}_4C_3$ for example, count down the rows: 0, 1, 2, 3, 4 and then across that row: 0, 1, 2, 3 which brings you to the second 4.

$$\text{Hence } {}_4C_3 = 4$$

The Binomial Theorem

(another use of combinations - just grab your combination numbers off Pascal's Triangle)

$$(x + y)^n = \sum_{k=0}^n {}_n C_k x^{n-k} y^k$$

Example:

$$\begin{aligned}(a - 2b)^5 &= {}_5 C_0 a^5 (-2b)^0 + {}_5 C_1 a^4 (-2b)^1 + {}_5 C_2 a^3 (-2b)^2 \\ &\quad + {}_5 C_3 a^2 (-2b)^3 + {}_5 C_4 a^1 (-2b)^4 + {}_5 C_5 a^0 (-2b)^5 \\ &= 1a^5 + 5a^4(-2b) + 10a^3(4b^2) + 10a^2(-8b^3) + 5a(16b^4) + 1(-32b^5) \\ &= a^5 - 10a^4b + 40a^3b^2 - 80a^2b^3 + 80ab^4 - 32b^5\end{aligned}$$

Congratulations!

If you know everything on both of these Pre-Calc power points,
then you are ready for:
your final exam
and
Calculus

If you don't know it all, then you need to practice some problems in your texts
and run these power points again.

